Statistical Properties of the Acoustic Field in Inhomogeneous Oceanic Environments: Scattering Matrix Approach

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LONG-TERM GOALS

To develop an effective method of description of statistical properties of acoustical signals and calculation of false alarm rate for a given probability distribution of locations of acoustic source(s) in space.

OBJECTIVES

- To develop an effective numerical algorithm of calculation of second statistical moments of the acoustic signals measured by a set of receivers located on sea bottom or arbitrarily distributed in space for inhomogeneous ocean waveguide (including the case of uneven bottom) in terms of probability distributions of source location.

- To investigate probability distributions of acoustical signals for typical environments including both deep water and littoral cases.

- To develop a numerical algorithm for predicting statistical moments of acoustic signals in underwater waveguides with horizontally-inhomogeneous and time-dependent parameters.

APPROACH

To quantify uncertainties in the acoustic field and associated probabilities of the detection and false alarm rate we use concept of a scattering matrix (SM) for acoustic modes. SM describes the process of transformation of modes when propagating through inhomogeneous region. SM depends on the parameters of the inhomogeneities but does not depend on coordinates of the source and receivers. Thus the dependencies on the environmental parameters and positions of the source and receivers are factored out and can be studied separately. In particular, if the SM is exactly known and position of the source is unknown, uncertainties of the detection can be expressed in terms of convolution of the SM with assumed probability distribution of the source location. On the other hand, if SM is not exactly known, additional uncertainty in source detection can be expressed in terms of statistical moments of SM itself.

Application of the concept of SM to characterize uncertainty associated with solitons of internal waves in the littoral areas is illustrated in Fig. 1.

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14. ABSTRACT
To develop an effective method of description of statistical properties of acoustical signals and calculation of false alarm rate for a given probability distribution of locations of acoustic source(s) in space.

15. SUBJECT TERMS

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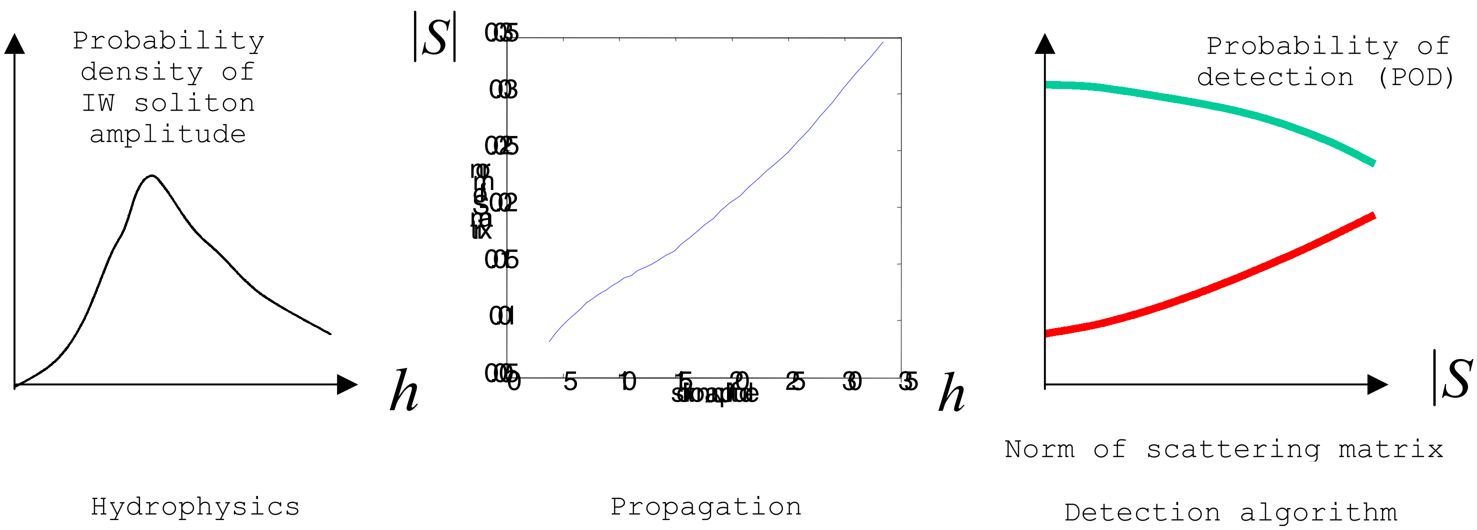

$$\langle POD \rangle = \int POD(|S|(h)) \cdot P_{sol}(h)\, dh$$

$$\langle FAR \rangle = \int FAR(|S|(h)) \cdot P_{sol}(h)\, dh$$

Figure 1. Application of a concept of scattering matrix to scattering by IW solitons [Graph: Plots of the probability density distribution for IW solitons, norm of scattering matrix as a function of IW soliton amplitude, and POD and FAR as a function of scattering matrix norm. Expressions for average POD and FAR.]

Calculation of the probability of detection (POD) and false alarm rate (FAR) in this case is separated into to three tasks, which are independent to a significant degree. First, probability distribution of the amplitudes of the IW solitons should be obtained. This task belongs to the realm of hydrophysics. The next task is to build SM and calculate its characteristics (e.g., a norm of SM, or a few largest eigenvalues) as function of soliton amplitude. This is purely sound field computation problem. Third, the dependencies of POD and FAR on the chosen SM characteristics should be calculated. These dependencies are determined by a configuration of a concrete detection system and detection algorithm used. They can also be obtained by numerical simulations. Finally, average POD and FAR values (or different statistical moments of POD and FAR) can be calculated by straightforward averaging.

WORK COMPLETED

The description of the effects of ocean inhomogeneities based on the concept of SM was applied to the case of sound propagation through perturabations due to internal wave solitons in shallow water. SM was calculated in the Born approximation. Its spectrum and appropriate eigenvectors were also computed for different soliton amplitudes.

Hydrodynamic description of the IW solitons for a case of two-layer fluid with constant Brunt-Vaisala frequencies separated by a density jump was obtained. This model is not restricted to the case of relatively weak KdV solitons and allows consideration of the "strong" solitons with amplitudes comparable with the layers thickness.

Equations governing the first and the second statistical moments of the acoustic field were developed in the Markov approximation. Preliminary version of the diffusion approximation of those equations was obtained, and the matrix of diffusion coefficients was calculated. The results for RMS of horizontal refraction angle were compared with the experimental data (obtained in the framework of another ONR-sponsored project).

RESULTS

Fig.2 illustrates a model of the shallow water waveguide and IW soliton used in calculations. IW soliton propagates at some angle α with respect to acoustic path. In a particular example below it was chosen $\alpha = 80^0$. Since soliton profile is assumed to be the same across propagation direction this angle is kept constant.

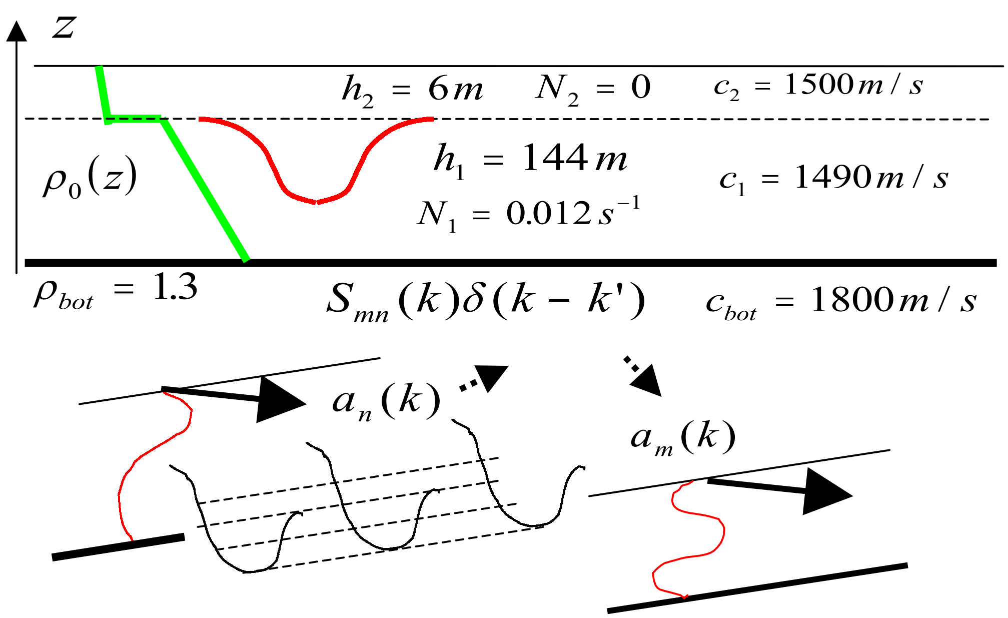

Figure 2. A model of a waveguide accepted for numerical simulations.
[Graph: Two-layer fluid with 6 m thick upper homogeneous layer and 144 m lower layer with constant buoyancy frequency. Illustration of acoustic modes scattering by a soliton.]

Frequency of the signal was 100 Hz. Sixteen acoustic modes were included into calculations. Spectrum of the SM and appropriate eigenvectors are shown on Fig. 3. One can see, that there are only three significant eigenvalues. This means that there are only three particular configurations of the acoustic field (i.e., superposition of the acoustic modes with specific amplitudes determined by appropriate eigenvectors) that are significantly affected by IW soliton. This information allows to tune a detection algorithm in such a way that those configurations of the field will have the smallest impact.

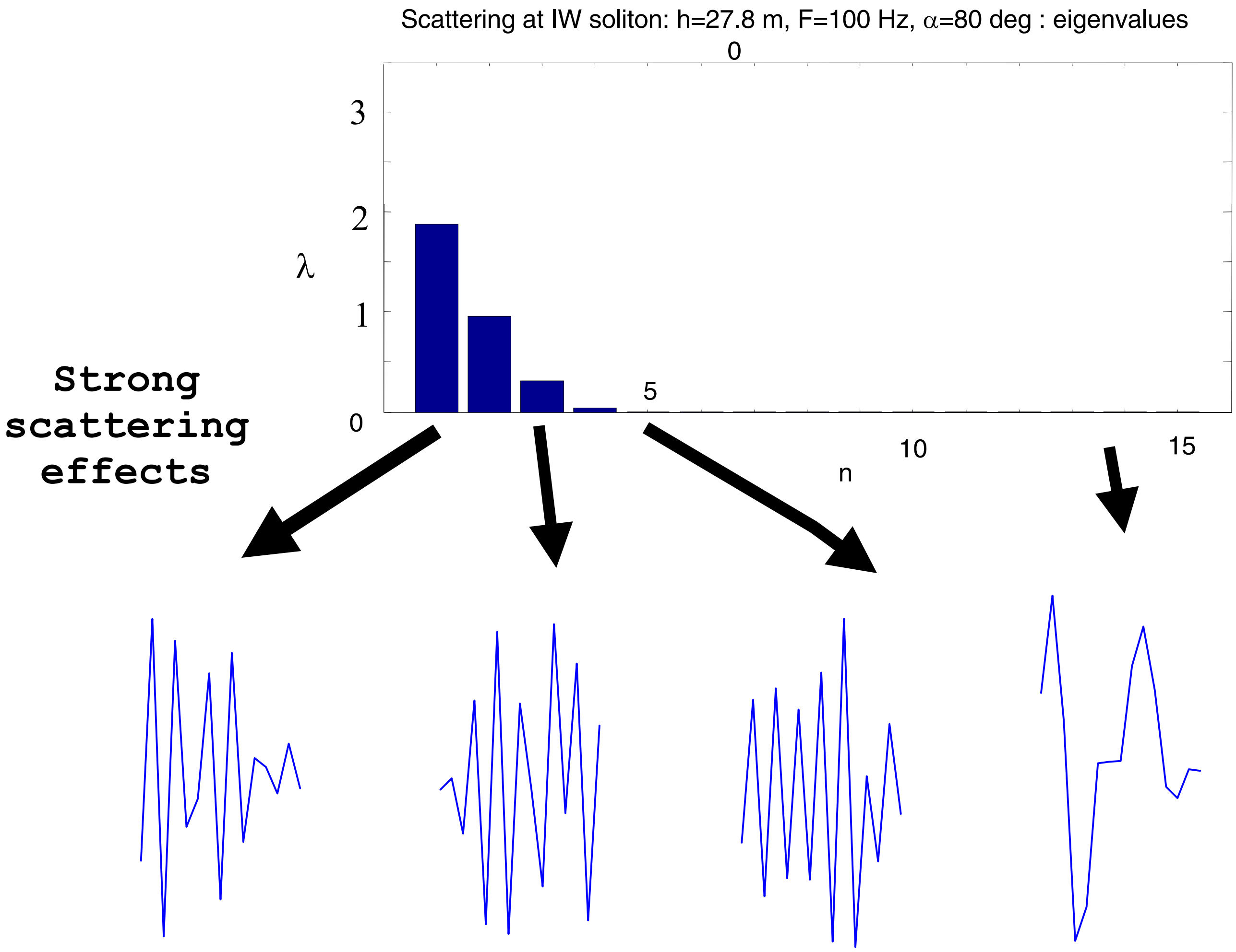

Figure 3. A model of a waveguide accepted for numerical simulations.
[Graph: Spectrum of SM as a function of eigenvalue index rapidly decreases. Eigenfunctions corresponding to three largest eigenvalues and a small one are shown on the lower panel.]

The next group of the results concerns calculation of the statistical moments of the field. Basic governing equations were obtained in the Markov approximation. For the average field and for the second moments B_{nm} the equation read:

$$\bar{a}_n(k,x+\Delta x) = \bar{a}_n(k,x)\exp\left(i\xi_n\Delta x\right) + \Delta x\sum_m \bar{S}_{nm}(k)\bar{a}_m(k,x)$$

$$\frac{B_{nn'}(k,x+\Delta x) - B_{nn'}(k,x)}{\Delta x} = B_{nn'}(k,x)\frac{\exp\left[i\left(\xi_n - \xi_{n'}\right)\Delta x\right] - 1}{\Delta x} +$$

$$+\sum_{m'}\exp\left(i\xi_n\Delta x\right)\bar{S}^*_{n'm'}(k)B_{nm'}(k,x) + \sum_{m'}\exp\left(-i\xi_{n'}\Delta x\right)\bar{S}_{nm}(k)B_{mn'}(k,x) +$$

$$+\Delta x\sum_{m,m'}\bar{S}_{nm}(k)\bar{S}^*_{n'm'}(k)B_{mm'}(k,x) + \sum_{m,m'}\int dk' E_{nn',mm'}(k,k')B_{mm'}(k',x)$$

The expressions for the average scattering matrix $\overline{S}_{nm}$ and scattering cross-sections $E_{nn',mm'}$ were obtained. Those equations are not very well suited for immediate numerical implementation because of the dependency of cross-component of the wave-vector *k* and integral term in the second equation. Further simplification of this equation will be introduced which is based on the narrowness of the scattering diagram which will allow to introduce diffusion approximation with respect to horizontal angles.

The third group of the results is related to development of hydrodynamic model of IW solitons. Existing models usually consider weakly-nonlinear IW case where the solitons are described by KdV equation. However, this model is not applicable to the cases where soliton amplitude is comparable to or even bigger then thickness of fluid layers (Stanton and Ostrovsky, 1998). Such solitons are often observed in the littoral areas. A “2.5-layer model” was suggested for description of such strong solitons. Stratification was assumed to consist of two layers with constant buoyancy frequency separated by a density jump. The results of numerical simulation performed for the case of IW observed during COPE experiment (Stanton and Ostrovsky, 1998) were compared with experimental data. The results appeared to be in a good agreement.

IMPACT/APPLICATIONS

This work should have an impact on design of detection algorithm. The algorithms should be made insensitive to the components of the acoustic field which are most affected by the unknown inhomogeneities of the ocean waveguide associated, e.g. with IW solitons or bottom profile features. Those field configurations are described by eigenvectors of scattering matrix. The results obtained will allow to quantify the effects of uncertainties in the description of oceanic environment on probability of detection and false alarm rate.

TRANSITIONS

None.

RELATED PROJECTS

Experimental Verification of a Horizontal-Refraction-Tomography Technique Using North Pacific Acoustic Laboratory Data (N00014-02-IP2-0035).

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